Decision Analysis Linguistic Framework

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Abstract. Everyday human beings are faced with situations they should choose among different alternatives by means of reasoning and mental processes when solving a problem. Many of these decision problems are under uncertain environments including vague, imprecise and subjective information that is usually modeled by linguistic information due to the use of natural language and its relation to mental reasoning processes of the experts when expressing their judgments. In a decision process multiple criteria can be evaluated which involving multiple experts with different degrees of knowledge. Such process can be modeled by using Multi-granular Linguistic Information (MGLI) and Computing with Words (CW) processes to solve the related decision problems. Different methodologies and approaches have been proposed to accomplish this process in an accurate and interpretable way. In this paper we propose a useful Decision Analysis Framework to manage this kind of problems by using the Extended Linguistic Hierarchy (ELH), 2-tuples linguistic representation model and its computational method. The developed Framework has many advantages when dealing with a complex problem in a simple way and its capability of having easy and useful reasonably results.

Keywords: 2-tuple Linguistic Model, Decision Analysis Framework, Fuzzy Linguistic Approach, Extended Linguistic Hierarchy

1 Introduction

In most of their day-to-day activities human beings are constantly making decisions. The multiple facets of real world decision problems are well addressed by Multi-Criteria Decision Making (MCDM) [1]. The crucial point of interest within the MCDM is the analysis and the modelling of the multiple decision makers' preferences giving rise to Multi-Expert Decision Making (MEDM). For many researchers the study of the decision-making processes has always been a field of great interest [2][3][4][5]. Besides, the mentioned process involves all kind of organizations; these must take decisions in order to survive in a dynamic environment [3].

To evaluate decision situations, there are contexts in which information cannot be assessed precisely in a quantitative form but it may be measured in a qualitative one, thus, the experts, that are involved in the making decision process, must deal with vague, imprecise and probably incomplete information. In these situations, information is normally modeled by using a *linguistic approach* [6][7][8] allowing the experts to express their opinions with words rather than numbers (e.g. when evaluating the *comfort* or *design* of a car, terms like *good*, *medium*, *bad* can be used).

Therefore, the linguistic approach is a technique that represents qualitative information as linguistic values by means of *linguistic variables* [6], that is, variables whose values are not numbers but words or sentences in a natural language. Each linguistic value is characterized by its *syntax* (label) and *semantic* (meaning). The label is a word or a sentence belonging to a linguistic term set and the meaning is a fuzzy subset in a universe of discourse. The concept of linguistic variables provides an estimated measure since words are less precise than numbers. This is more effective because the experts may feel more comfortable using words they really know and understand in accordance with the context of use of these words. Also, when offering different expression domains or different linguistic term sets (multi-granular information) to the experts, this solution would be suitable to adjust the degree of experience of each one. Therefore, this will prevent from loosing information when considering just one expression domain for all the experts since many of them may need a larger expression domain than others according to their knowledge [9][10].

Linguistic Decision Analysis (LDA) is based on the use of linguistic approach and is applied for solving decision making problems under linguistic information. In the literature, many applications of linguistic decision analysis may be found in order to solve real world activities for instance group decision making, MCDM, sensory evaluation, Human Resources evaluation, networking decision analysis, recommendation models and the list goes on [10][11][12][13][14][15][16]. As generalization, LDA is a process that is composed of different phases such as: definition problem, information gathering, computation and finally presentation of results in a suitable way.

In this paper, we focus on complex decisions under uncertainty showing a framework to be analyzed with multiple experts and multiple criteria using multi-granular linguistic information. Consequently, this paper is organized as follows. Section 2 reviews basic concepts about linguistic background that the framework will be used to model uncertain information and multi-granular information. Section 3 presents the linguistic framework, its application and phases in order to analyze decisions. Then, section 4 proposes an example of use applied in recruitment process. Finally, Section 5 shows some conclusions and future work.

2 Linguistic Background

In the real world there are many situations in which problems must manage with vague and imprecise information that usually involves uncertainty. In those cases in which the uncertainty is not of probabilistic nature, it is complex to provide numerical precise information when the knowledge is vague. In these problems is more adequate that the involved experts provides linguistic descriptors to express their assessments due to the fact that the use of linguistic terms in problems with non-probabilistic uncertainty has produced successful results in different fields.

When the decision analysis depends highly on subjective, vague and ill-structured information must have a model to manage this kind of information. Therefore, we consider the use of the fuzzy linguistic approach [6] to model and manage the inherent uncertainty in this kind of problems and the 2-tuple linguistic model to represent linguistic information [17]. Additionally, proposal framework offers multiple linguistic scales (multi-granular information) giving more flexibility to the different experts involve in the problem and, to manage this, we use Extended Linguistic Hierarchies (ELH) method. For this reason, in this section we review in short the concepts and methods used in the proposed Framework such as the fuzzy 2-tuple linguistic model, extended linguistic hierarchies and his computational method.

2.1 The 2-tuples linguistic model

This model was presented in [18], for overcoming the drawback of the loss of information presented by the classical linguistic computational models: (i) The semantic model [19], (ii) and the symbolic one [20]. It is based on the symbolic method and takes as the base of its representation the concept of Symbolic Translation.

Definition 1. *The Symbolic Translation of a linguistic term $s_i \in S = \{s_0, ..., s_g\}$ is a numerical value assessed in $[-0.5, 0.5)$ that supports the "difference of information" between an amount of information $\beta \in [0, g]$ and the closest value in $\{0, ..., g\}$ that indicates the index of the closest linguistic term in $S\,(s_i)$, being $[0, g]$ the interval of granularity of S.*

From this concept a new linguistic representation model was developed, which represents the linguistic information by means of a linguistic 2-tuple. It consists of a pair of values namely, $(s_i, \alpha) \in \overline{S} \equiv S \times [-0.5, 0.5)$, being $s_i \in S$ a linguistic term and $\alpha \in [-0.5, 0.5)$ a numerical value representing the symbolic translation. This representation model defined a set of transformation functions between numeric values and linguistic 2-tuples to facilitate linguistic computational processes.

Definition 2. *Let $S = \{s_0, ..., s_g\}$ be a linguistic terms set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation. The 2-tuple set associated with S is defined as $\overline{S} = S \times [-0.5, 0.5)$. A 2-tuple that expresses the equivalent information to β is then obtained as follow:*

$$\Delta : [0, g] \rightarrow \overline{S}$$

$$\Delta(\beta) = (s_i, \alpha), with \begin{cases} s_i, \; i = round(\beta) \\ \alpha = \beta - i, \;\; \alpha \in [-0.5, 0, 5) \end{cases} \tag{1}$$

being round $(\cdot)$ the usual round operation, i the index of the closest label, s_i, to "β", and "α" the value of the symbolic translation.

It is noteworthy to point out that Δ is a one to one mapping and $\Delta^{-1} : \overline{S} \rightarrow [0, g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$. In this way the 2-tuple of $\overline{S}$ is identified by a numerical value in the interval $[0, g]$.

Remark 1. The transformation of a linguistic term into a linguistic 2-tuples consists of adding value 0 as symbolic translation: $s_i \in S \Rightarrow (s_i, 0) \in \overline{S}$. On other hand, $\Delta(i) = (s_i, 0)$ and $\Delta^{-1}(s_i, 0) = i$, $\forall i \in \{0, 1, .., g\}$.

If $\beta = 3.25$ is the value representing the result of a symbolic aggregation operation on the set of labels, $S = \{s_0 = Nothing, s_1 = VeryLow, s_2 = Low, s_3 = Mediums, s_4 = High, s_5 = VeryHigh, s_6 = Perfect\}$, then the 2-tuple that expresses the equivalent information to β is $(medium, .25)$. See Fig. 1.

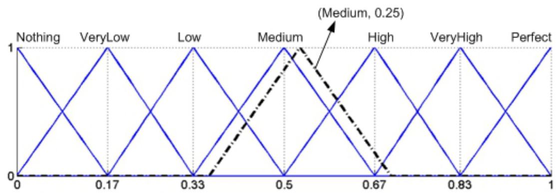


Fig. 1. 2-tuple linguistic representation

This model has a linguistic computational technique based on the functions Δ and Δ^{-1}, for a further detailed description see Ref. [21].

2.2 Extended Linguistic Hierarchies

Another important aspect related to the linguistic information is the granularity of uncertainty, i.e., the level of discrimination among different degrees of uncertainty. When an expert has more knowledge about the problem, he/she needs more granularity in the linguistic scales to express their assessments , i.e, a linguistic scale with a higher number of linguistic terms. Typical values of cardinality used in the linguistic models are odd ones, such as 5, 7 or 9, being 5 an adequate granularity for an expert with a low level of knowledge about the problem and 9 for an expert with a high level of knowledge.

The proposed decision support Framework offers to the experts a flexible expression domain with several linguistic scales to express their assessments according to their degree of knowledge about the problem. The greater the knowledge, experience or skill of an expert then the greater its ability in assessments, therefore he/she will use an expression domain with greater granularity.

Different approaches dealing with multi-granular linguistic information have been proposed. In the proposed Framework shall use the ELH [22] approach to model and manage multi-granular linguistic information because of its features of flexibility and accuracy in the processes of computing with words (CW) in multi-granular linguistic contexts. An ELH is a set of levels, where each level represents a linguistic term set with different granularity from the remaining levels of the ELH. Each level belongs to an ELH is denoted as $l(t, n(t))$ being t a number that indicates the level of the ELH and $n(t)$ the granularity of the terms set of the level t. To build an ELH have been proposed a set of extended hierarchical rules:

1. Rule 1: A finite set of levels, $l(t, n(t))$ with $t = 1, .., m$, that defines the multi-granular linguistic context required by experts to express their assessments are included.
2. Rule 2: to obtain an ELH a new level, $l(t^*, n(t^*))$ with $t^* = m + 1$, should be added. This new level must have the following granularity:

$$n(t*) = (L.C.M.(n(1) - 1, ..., n(m) - 1)) + 1 \tag{2}$$

   being L.C.M. the Least Common Multiple.

ELH building process then consists of two processes: i) It adds m linguistic scales used by the experts to express their information. And ii) then it adds the term set $l(t*, n(t*))$, with $t = m + 1$, according to Eq. (2). Therefore, the ELH is the union of all levels required by the experts plus the new level $l(t^*, n(t^*))$.

$$ELH = \bigcup_{t=1}^{t=m+1} (l(t, n(t)))$$

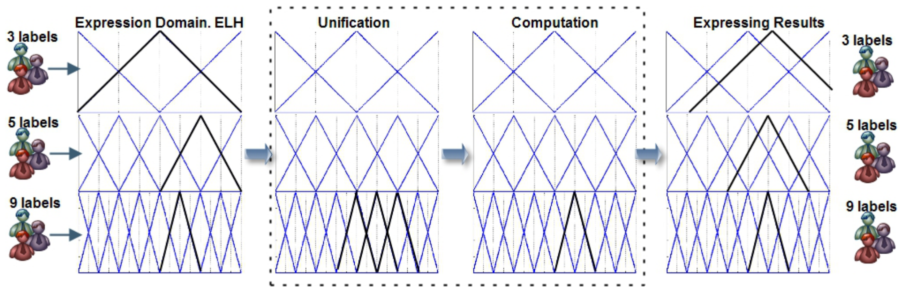


Fig. 2. CW process in ELH

The use of multi-granular linguistic information makes the processes of CW more complex. ELH computational model needs to make a three-step process.

1. Unification phase. The multi-granular linguistic information is conducted into only one linguistic term set, that in ELH is always $S^{n(t*)}$, by means of a transformation function $TF_b^a(\cdot)$:

Definition 3. *Let $S^{n(a)} = \{s_0^{n(a)}, ..., s_{n(a)-1}^{n(a)}\}$ and $S^{n(b)} = \{s_0^{n(b)}, ..., s_{n(b)-1}^{n(b)}\}$ be two linguistic term sets, with $a \neq b$. The linguistic transformation function is defined as:*

$$
\begin{aligned}
TF_b^a : \overline{S}^{n(a)} &\rightarrow \overline{S}^{n(b)} \\
TF_b^a\,(s_j^{n(a)}\,,\,\alpha_j^{n(a)}) \;&=\; \Delta_S\left(\frac{\Delta^{-1}(s_j^{n(a)}, \alpha^{n(a)}) \cdot (\,n(b) - 1\,)}{n(a)\,-\,1}\right) \\
&= \,(s_k^{n(b)}\,,\,\alpha_k^{n(b)})
\end{aligned}
\tag{3}
$$

2. Computational process. Once the information is expressed in only one expression domain $S^{n(t*)}$, the computations are carried out by using the linguistic 2-tuple model.
3. Expressing results. In this step the results might be transformed into any level, t, of ELH in a precise way by using Eq. (3) to improve the understanding of the results if necessary.

Remark 2. In the processes of CW with information assessed in an ELH, the linguistic transformation function, TF_b^a, performed in the unification phase, a, might be any level in the set $\{t = 1, .., m\}$ and the computational processes are carried out in the level b that it is always the level t^* (See Eq. (3)).

It was proved in [22] that the transformation functions between linguistic terms in different levels of the Extended Linguistic Hierarchy are carried out without loss information. Figure 2 shows the steps listed above in order to clarify the CW process in ELH.

3 Decision Analysis Linguistic Framework

The Decision Analysis Process, evaluated in Multi-Expert and Multi-Criteria contexts, requires a management tool to achieve a ranking of alternatives set in merit order. Besides, to consider the expert knowledge degree involved in such process we have proposed and modeled multi-granular linguistic information with ELH. To achieve this, in this section the main results of our research is introduced trough a Linguistic Decision Analysis Framework.
Figure 3 shows the framework phases graphically that are further detailed below.

Framework data definition. This phase defines the evaluation context in which the experts will express their preferences about the evaluated objects, e.g. each criterion in alternatives. At this point, the linguistic descriptors and their semantics are chosen as well as each alternative is identified as a solution of potential problem. It also determines the criteria to evaluate every alternative and the experts who are involved in the decision process. Given that, in the Linguistic Decision Analysis process take part a group of experts may happen that all experts agree in the expression domain either different experts may feel

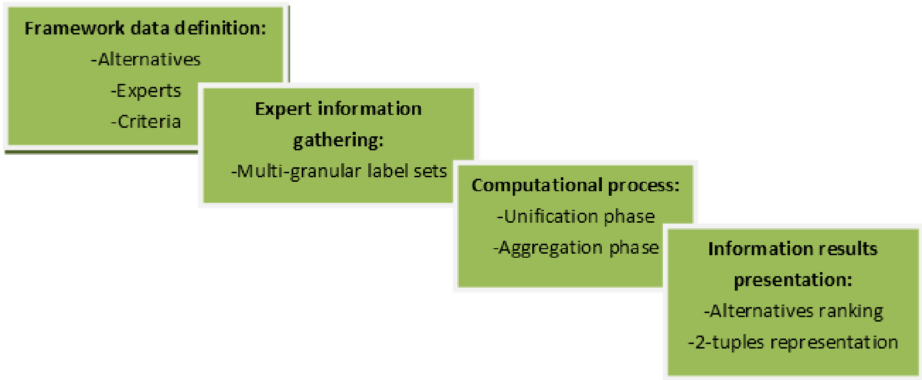


Fig. 3. Framework phases

better expressing their knowledge in a linguistic term set whilst other ones prefer a different linguistic term set to express their knowledge. Thus, linguistic terms sets are organized into an ELH and assigned to different experts. According to the above said, the framework data definition consists of:

- A finite set of alternatives $X = \{x_k, k = 1, ..., q\}$.
- A finite set of criteria $C = \{c_j, j = 1, ..., n\}$.
- A finite set of experts $E = \{e_i, i = 1, ..., m\}$ that express their assessments by using different linguistic scales of information in ELH.

Expert information gathering. Due to the fact that, the linguistic decision analysis used in this framework is based on the MCDM problems. The experts provide their knowledge in utility vectors that contain a linguistic assessment for each criterion of the evaluated alternatives. Thereby, experts express their assessments on every criterion considering every alternative using their linguistic term set. For example, let consider expert e_i who has associated the linguistic term set $S_i^{n(r)} = \left\{S_0^{n(r)}, ..., S_{n(r)-1}^{n(r)}\right\}$ with granularity $n(r)$ in the r level of ELH. This is used by the expert e_i to give his assessments to each $c_j \in C$ in $x_k \in X$. Let $U_i = \left\{u_{11}^i, ..., u_{1n}^i, u_{21}^i, ..., u_{2n}^i, ..., u_{m1}^i, ..., u_{mn}^i\right\}$ be a vector of preferences given by expert e_i and $u_{kj}^i \in S_i^{n(r)}$ the expert's assessment for the criterion c_j in alternative x_k and $S_i^{n(r)} \in$ ELH. Due to the fact that the Framework will use the linguistic 2-tuple computing model the linguistic preferences provided by the experts will be transformed into linguistic 2-tuples according to the Remark 1.

Computational process. In this phase linguistic utility vectors provided by the experts and transformed into linguistic 2-tuples will be used in processes of computing with words in order to rate each alternative. It consists of two steps:

- Unification step. All the assessments provided by the experts in different linguistic scales are transformed in a unique expression domain, so called t^* whose granularity is given by the Eq. (2). Thus, transformation must be the last level of the ELH according to Eq. (3). Once the information has been unified, will be expressed by means of linguistic 2-tuples in $S^{n(t^*)}$.
- Aggregation step. In order to obtain the global assessments for each alternative the information must be aggregated. In literature there is a considerable research related to aggregation operators [23][24][25] and depending on the problem different types of aggregation operators can be used. In this framework two of them are used to aggregate experts assessments, Linguistic aggregation operators of non-weighted information (Geometric Mean) and Linguistic aggregation operators of weighted information (Weighted Aggregation Operator).

Definition 4. *Let* $((l_1, \alpha_1), .., (l_m, \alpha_m)) \in \overline{S}^m$ *be a 2-tuples linguistic vector, geometric mean operator is defined as follows:* $G : \overline{S}^m \rightarrow \overline{S}$

$$G : [((l_1, \alpha_1), .., (l_m, \alpha_m))] = \left[\prod_{i=1}^{m} \Delta^{-1} [(l_i, \alpha_i)] \right]^{\frac{1}{m}} = \left[\prod_{i=1}^{m} \beta_i \right]^{\frac{1}{m}} \tag{4}$$

With the Geometric Mean operator the linguistic information provided by different sources is equal importance, i.e., all sources are equally important in the aggregation process. However, in some cases, a rational assumption about the resolution process could be associating more importance to the experts who have more "knowledge" or "experience". These values can be interpreted as *importance degree*, *competence*, *knowledge* or *ability* of the experts. In addition some experts could have some difficulties in giving all their assessments due to lack of knowledge about part of the problem. Besides the use of different scales, the expert should be carried out in different way with weighted aggregation operator.

Definition 5. *Let* $((l_1, \alpha_1), .., (l_m, \alpha_m) \in \overline{S}^m$ *be a vector of linguistic 2-tuples, and* $w = (w_1, ..., w_m) \in [0, 1]^m$ *be a weighting vector such that* $\sum_{i=1}^{m} w_i = 1$*. The 2-tuple aggregation operator associated with* w *is the function* $G^w : \langle \overline{S} \rangle^m \rightarrow \langle \overline{S} \rangle$ *defined by*

$$G^w[(l_1, \alpha_1), .., (l_m, \alpha_m)] = \Delta_{\overline{s}} \left(\sum_{i=1}^{m} w_i \beta_i \right) \tag{5}$$

In our Framework , this step consists of two additional steps:
Computing experts collective criteria values. It is a function $G^w_{jk} : \overline{S} \rightarrow S$ for each criterion c_j in each alternative x_k.
Computing global value. It consists of aggregating all criteria for each alternative to compute global value, $G : S^p \rightarrow S$ using arithmetic mean aggregation operator.

Definition 6. *Let* $((l_1, \alpha_1), .., (l_n, \alpha_n)) \in \overline{S}^n$ *be a 2-tuples linguistic vector, arithmetic mean operator is defined as follows:* $\overline{x^e} : \overline{S}^n \rightarrow \overline{S}$

$$\overline{x^e}[(l_1, \alpha_1), .., (l_n, \alpha_n)] = \Delta(\sum_{i=1}^{n} \frac{1}{n} \Delta^{-1}(r_i, \alpha_i)) = \Delta(\frac{1}{n} \sum_{i=1}^{n} \beta_i) \tag{6}$$

Information results presentation. Results are presented in two different complementary ways. First of all, an alternatives ranking ordered by preference is showed. The second presentation uses 2-tuples representation to express the alternatives ranking into a particular linguistic scale for a specific expert.

4 Illustrative Example

In this section, we show an application in a complex decision problem with the aim to highlight the usefulness and effectiveness of the Decision Analysis Linguistic Framework proposed. To do so, we introduce the problem context and then we apply the framework phases.

Let consider a medium sized organization called "Argentic" which operates on a nationally scale, is dedicated to manufacture advanced communication devices for complex environments. After a very difficult beginning, the company managed to stabilize and started to grow in the market.

Considering that the human resource is one of the most important aspect of any organization, the most important decisions to be taken are related to hiring new staff. For this particular example, we will consider the decision making process related to incorporating new staff for the post of *director of technologies*.

In this illustrative example, the framework is composed by 5 candidates $X = \{A1, A2, A3, A4, A5\}$. The new job involves understanding global and specific aspects of the organization. Therefore, it is necessary to take into account certain issues such as the *accomplish of some organizational objectives*, the *ability to work with other people* as well as *their motivation*, the *ability to understand problems and analyze relevant matters related to them* and finally *working in a proactive manner* . Therefore, each candidate is evaluated according to these 5 criteria $C = \{c_1, c_2, c_3, c_4, c_5\}$.

Given these crucial aspect that are involved in the selection of Human Resources, it would be necessary to consider the opinion of three experts $E = \{e_1, e_2, e_3\}$. The first expert, e_1, who provides the overall view of the organization is the CEO. Experts e_2 and e_3 correspond respectively to the head of the Research Department and the head of the Infrastructure Department in order to provide an operative and technical point of view. The fixed linguistic terms set have been 9 labels (S_1)for the CEO e_1, and 7 labels (S_2) for the heads of the department e_2 and e_3.

- S_1: *Very low(VL), Low(L), Medium low(ML), Medium(M), Medium high(MH), High(H), Very high(VH)*
- S_2: *Nothing(N), Very low (VL), Low (L), Medium low (ML), Medium (M), Medium high (MH), High (H), Very high(VH), Perfect(P).*

Therefore, the ELH composed of the following three levels: $l(1, 7)$, $l(2, 9)$ and $l(3, 25)$, being this last level denoted by $l(t*, n(t*))$. In this qualitative framework, the preferences provided by the experts are showed in Table 1:
The evaluators' preferences are transformed into the last level of the ELH in the level $t = 3$ by means of the transformation functions, TF_3^1 and TF_3^2. We

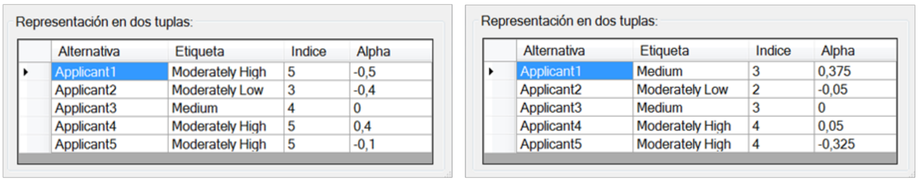
Table 1. Information gathering for experts

e_1	A1	A2	A3	A4	A5	e_2	A1	A2	A3	A4	A5	e_3	A1	A2	A3	A4	A5
c_1	VL	VL	H	VH	VH	c_1	ML	VL	VH	H	H	c_1s	L	ML	H	M	H
c_2	H	H	VL	M	VL	c_2	H	MH	L	MH	ML	c_2	VH	H	ML	MH	ML
c_3	VH	M	VL	H	VH	c_3	M	L	VL	H	H	c_3	MH	VL	VL	VH	VH
c_4	M	VL	P	VL	M	c_4	VL	L	H	L	ML	c_4	L	L	H	ML	ML

use the *linguistic 2-tuple weighted average operator* to aggregate the preferences with the following weight vector:

$$W = (0.4, 0.3, 0.3)$$

The global values can be expressed in any linguistic term set of the ELH by means of the transformation functions, TF_1^3 and TF_2^3. Therefore, alternatives are described with labels and the "symbolic translation"(i.e. 2-tuple representation) in the selected level of an ELH. In this illustrative example, two levels are considered, Figure 4 shows global values in S^9 and S^7, respectively. ω w


Representación en dos tuplas:

Alternativa	Etiqueta	Indice	Alpha
Applicant1	Moderately High	5	-0,5
Applicant2	Moderately Low	3	-0,4
Applicant3	Medium	4	0
Applicant4	Moderately High	5	0,4
Applicant5	Moderately High	5	-0,1

Representación en dos tuplas:

Alternativa	Etiqueta	Indice	Alpha
Applicant1	Medium	3	0,375
Applicant2	Moderately Low	2	-0,05
Applicant3	Medium	3	0
Applicant4	Moderately High	4	0,05
Applicant5	Moderately High	4	-0,325

Fig. 4. 2-tuple global values for the candidates

Also, the framework has the option of displaying the results as a global ranking for alternatives ordered by its importance (for all experts) and finally, the results of the evaluations of each experts considering just his own assessments.

Figure 5 illustrates these results, on the left side global values and on the right the individual rankings. This framework and the results shown in this paper have been implemented and now we are working on extending the results shown here.

5 Conclusions and Future Works

Decision Analysis Process becomes complex in environments where uncertainty is high, and it can be better performed with a linguistic approach allowing the experts to work with well-known linguistic term sets. Thus, in this paper, a powerful and flexible tool to manage multi-criteria, multi-expert and multi-granular decision making processes is presented to get overall results as an alternative to

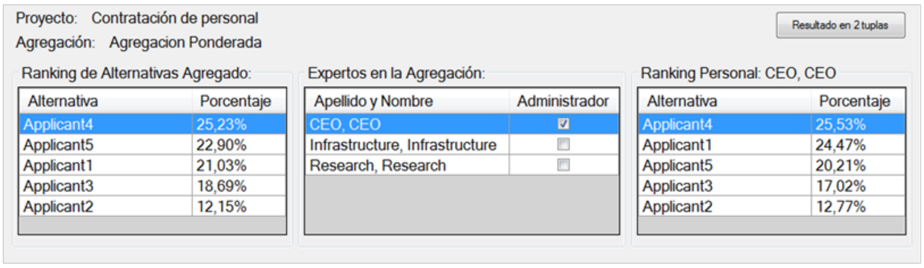


Fig. 5. Ranking of the candidates

solve this problem. Besides, the global solution must be reached without losing information but also taking into account the particular nature of the criteria and the specific differences among the experts through aggregation processes.

The proposed framework is computationally complex but this difficulty is not reflected on those who are involved in the decision making process. For the experts such complexity does not exist and its interface is easy, simple and quick which is separated in well-defined phases. Finally, we have shown an illustrative example to prove its use, flexibility as well as benefits.

Currently, the computation capability is expanded by using different aggregation operators such as Ordered Weighted Averaging (OWA) aggregation operators family. In addition, we are comparing different methodologies and decision making approaches such as Analytic Hierarchy Process (AHP).

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