

Polyhedral studies on vertex coloring problems

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Many variants of the vertex coloring problem have been defined, such as *precoloring extension* [1], μ -coloring [2], (γ, μ) -coloring [3], and *list coloring* [7]. These problems are NP-hard, as they generalize the classical vertex coloring problem [6]. On the other side, there exist several families of graphs for which some of these problems can be solved in polynomial time [3, 4]. The standard integer programming model for coloring problems uses a binary variable x_{vc} for each vertex v and each color c to indicate whether v is assigned c or not. An extension of this model considers binary variables w_c for each color c to indicate whether color c is used or not [5]. In this work we study this formulation for the polynomial cases of the coloring problems mentioned above. In particular, we prove that if the classical vertex coloring problem yields an integer polytope for a family of graphs, then the same holds for μ -coloring, (γ, μ) -coloring, and list coloring over the same family. We prove that the polytope associated to these problems over trees is integer and that adding the clique inequalities [5], the resulting polytope is integer over block graphs also. Finally, we give a new family of facet-inducing valid inequalities for the standard formulation and we provide empirical evidence suggesting that this family completely describes the polytope associated to these problems over cycles (and probably cactii graphs).

References

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